International Journal of Theoretical Physics, Vol. 30, No. 9, 1991

Nonlinear Electromagnetic Effects Induced by Torsion

L. C. Garcia de Andrade[1] and J. U. Cinelli L. de Oliveira[1]

Received October 29, 1990

Nonlinear electromagnetic effects induced by massive electrodynamics with torsion are discussed. The charge is shown to be conserved in the case the torsion vector is divergence-free. A torsion-noise effect is suggested.

1. INTRODUCTION

Recently Garcia de Andrade (1990) suggested an interaction Lagrangian term in Cartan space given by

$$\mathscr{L}_I = \lambda R(\Gamma) A^2 \tag{1}$$

where Γ is the Riemann–Cartan U_4 space connection, $\Gamma^i{}_{jk} = \{^{\,i}_{jk}\} - K^i{}_{jk}$, where $\{\cdot\}$ are the Christoffel symbols for Riemannian metric g_{ij} $(i, j = 0, 1, 2, 3)$, $A^2 \equiv A_i A^i$, where A^i is the electromagnetic four-vector potential, and $R(\Gamma)$ is the Ricci scalar in U_4. The symbol $K^i{}_{jk}$ represents the contortion of spacetime given in terms of torsion $\Gamma^i{}_{[jk]} = S^i{}_{jk}$ by

$$K^i{}_{jk} \equiv S^i_{jk} - S_{kj}{}^i + S^i{}_{jk}$$

The Lagrangian term (1) gives an interaction between a photon field and a torsion field in terms of a Proca field term. It also provides us with a non-gauge-invariant Lagrangian of the form

$$\mathscr{L} = \sqrt{-g}\,\{R(\Gamma)(1 + \lambda A^2) - \tfrac{1}{4} F_{ij} F^{ij} + J^i A_i\} + \mathscr{L}_{\text{matter}} \tag{2}$$

The main advantage of the interaction Lagrangian term (1) is that the photon feels torsion through a curvature term. The idea of nonlinear photons produced in high-curvature spacetimes has been previously discussed by Novello and Salim (1979) in the context of general relativity. Here we extend

[1]Grupo de Relatividade e Teoria do Campo, Instituto de Fisica, Universidade du Estado do Rio de Janeiro, Rio de Janeiro, RJ, Brazil.

0020-7748/91/0900-1259$06.50/0

the Novello–Salim results to include torsion effects and examine new effects such as polarization in vacuum induced by a propagating torsion field. We also show that a propagating torsion leads necessarily to a theory where the bosonic current (analogous to the fermionic current) is conserved. Finally, we argue that the massive photon field may be used as a source of the fifth force, since it has been shown by de Sabbata and Sivaram (1990) that the torsion induces a modification in the Newtonian force law of gravity. The coupling constant can be computed as

$$\lambda \cong m_\gamma^2 / \{m_P^2 c^2 (\nabla \cdot \mathbb{Q})\} \tag{3}$$

(Garcia de Andrade *et al.*, 1990), where m_γ is the photon mass and m_P is the Planck mass if the term $\nabla \cdot \mathbb{Q}$ can be computed from the torsion equation (R. Hammond and L. C. Garcia de Andrade, personal correspondence, 1989).

2. MAXWELL–PROCA–CARTAN ELECTRODYNAMICS

Application of the Euler–Lagrange equations for the field A^i leads to the Maxwell–Cartan–Proca field equations (Garcia de Andrade, 1990) and considering the weak torsion approximation,

$$\partial_{[i} F_{jk]} = 0 \tag{4}$$

$$\partial_i F^{ij} = 4\pi J^j - \frac{3\lambda}{k} (\partial_l Q^l) A^j \tag{5}$$

where λ is the coupling constant of the Lagrangian, $k = (8\pi G/c^4)$ is the Einstein constant, and Q^l are the components of the torsion vector of spacetime. Application of the partial derivative operator on both sides of equation (5) yields (where we adopt the gauge translational limit $g_{ij} \cong \eta_{ij}$ sometimes referred as Weitzenböck spacetime)

$$\partial_j \partial_i F^{ij} = \partial_j J^j - \frac{3\lambda}{k} (\partial_j \partial_l Q^l) A^j - \frac{3\lambda}{k} (\partial_l Q^l)\, \partial_j A^j \tag{6}$$

where we have used the Lorentz gauge $\partial_j A^j = 0$. Due to the symmetry properties of the term on the left-hand side, equation (6) reduces to

$$\partial_j J^j = \frac{3\lambda}{k} (\partial_j \partial_l Q^l) A^j \tag{7}$$

Expression (7) shows explicitly that the four-current vector J^i is not conserved unless the right-hand side vanishes. Now let us suppose that torsion propagates through a potential ϕ as $Q^l = \partial^l \phi$; substitution of this potential into (7) yields

$$\partial_j J^j = \frac{3\lambda}{k} A^j \partial_j \Box \varphi \tag{8}$$

which represents the derivative of bosonic current and where $\Box \equiv \partial_l \partial^l$ is the d'Lambertian wave operator. If the photon mass m_γ is constant, expression (8) reduces to

$$\partial_i J^i = 0 \tag{9}$$

which represents the conservation of the bosonic current associated to the photon mass. Another interesting application of the Maxwell–Proca–Cartan field equation (MPC) is the construction of a polarization vector induced by the torsion field $\mathbb{Q}$. To see how this can be done, let us consider the static limit ($A^0 = \Phi$, $\mathbb{A} = 0$) of equations (5). In terms of the electric and magnetic fields ($\mathbb{E}$, $\mathbb{B}$) we obtain the following form of MPC:

$$\nabla \cdot \mathbb{E} = 4\pi\rho - \frac{3\lambda}{k} q(\nabla \cdot \mathbb{Q}) \frac{e^{-\mu r}}{r} \tag{10}$$

$$\nabla \times \mathbb{E} = \mathbf{0} \tag{11}$$

where the Yukawa term in (10) comes from the solution $A^0 = (q/r)\, e^{-\mu r}$ of the Proca field equation $(\Box + m^2)A^0 = 0$ (Garcia de Andrade, 1990) in spacetimes with torsion. Let us now consider the electric field inside a polarized "medium" (vacuum with propagating torsion) given by

$$\mathbb{D} = \mathbb{E} + 4\pi\mathbb{P} \tag{12}$$

The Coulomb–Proca–Cartan equation (10) can be expressed in terms of a displacement vector $\mathbb{D}$ as

$$\nabla \cdot \mathbb{D} = 4\pi\rho' \tag{13}$$

where the new charge density can be expressed in terms of torsion (in the case of a point charge) by

$$\rho' = q\left[\delta(\mathbf{r}) - \frac{3\lambda}{k} \mathbb{Q} \cdot \nabla\left(\frac{e^{-\mu r}}{r}\right)\right] \tag{14}$$

where the polarization vector $\mathbb{P}$ is now given by

$$\mathbb{P} = \frac{3\lambda}{4\pi k} \frac{e^{-\mu r}}{r} \mathbb{Q} \tag{15}$$

where $\nabla \cdot \mathbb{E} = 4\pi\rho - \nabla \cdot \mathbb{P}$. From equation (15) we notice that in the absence of torsion vector $\mathbb{Q}$ the polarization vector $\mathbb{P}$ vanishes. However, there is also the possibility of polarization being induced by gravity in vacuum (de Sabbata and Gasperini, 1979). Besides the application considered in this paper, there is also the possibility of considering the contribution of the massive photons produced by torsion to contribute to the fifth force problem, since it induces a Yukawa-type force via torsion in much the same way in which the gravitational force is modified (de Sabbata and Sivaram, 1990).

3. DISCUSSION

Novello and Salim (1979) have discussed the analogy of an electric field inside a nonlinear dielectric in general relativity due to the dependence of the dielectric constant on light intensity. Analogous conclusions can be reached here in the case of nonlinear dielectrics in spaces with torsion. A detailed study along these lines will appear elsewhere. Also, in equation (8) the torsion potential φ is not necessarily harmonic, which gives the sort of torsion noise $\Box\varphi = \text{const}$. Further work in this direction is now underway.

REFERENCES

De Sabbata, V. (1988). In *Gravitational Measurements, Fundamental Metrology and Constants*, Venzo de Sabbata and V. N. Melnikov, eds., NATO Series C: Mathematical and Physical Sciences, Vol. 230.

De Sabbata, V., and Gasperini, M. (1979). *Lettere al Nuovo Cimento*, **24**, 15.

De Sabbata, V., and Gasperini, M. (1981). *Physics Letters*, **83A**, 115.

De Sabbata, V., and Gasperini, M. (1985). *Introduction to Gravitation*, World Scientific, Singapore.

De Sabbata, V., and Sivaram, C. (1990). *International Journal of Theoretical Physics*, **29**, 1.

Garcia de Andrade, L. C. (1990). *General Relativity and Gravitation*, **22**, 8.

Garcia de Andrade, L. C. (1991). Nonlinear electrodynamics and torsion, *Nuovo Cimento B*, to appear.

Garcia de Andrade, L. C., Oguri, V., Lopes, M., and Hammond, R. (1990). On the energy splitting of spectral lives, *Nuovo Cimento B*, submitted.

Hojman, S., Rosebaum, M., Ryan, M. P., and Shepley, L. C. (1978). *Physical Review D*, **17**, 3141.

Landau, L., Lifchitz, I., and Pitaevskii, L. P. (1984). *Electrodynamics of Continuous Media*, 2nd ed., Pergamon Press, Oxford.

Novello, M., and Salim, J. M. (1979). *Physical Review D*, **20**, 377.